

# Indirect Bounds on $Z \rightarrow \mu e$ , and Lepton Flavor Violation at Future Colliders

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## Abstract

Motivated by the interest in lepton number violating processes, we study the connection between the rate of  $Z \rightarrow \mu e$  decay and those of the low-energy processes  $\mu \rightarrow 3e$ , and  $\mu \rightarrow e$  conversion in nuclear field. We show that if the vector or axial  $Z$  form factors are dominant,  $Br(Z \rightarrow \mu e)$  is not observable, while if the  $Z$  dipole form factors are dominant, the relatively weak indirect bound  $Br(Z \rightarrow \mu e) < 7 \times 10^{-9}$  does not fully preclude a signal at future colliders, as TESLA. We finally comment on the relation of  $Z \rightarrow \mu e$  with  $Z \rightarrow \tau e$  and  $Z \rightarrow \tau \mu$  decays, and suggest a simple scaling law for these three processes.

## 1 Introduction

There are convincing evidences that neutrinos are massive and oscillate in flavor. This has as natural consequence an increased interest in lepton flavor violating processes, which is testified by a number of theoretical studies—see for instance [1]. Several experiments that may considerably improve on lepton flavor violating processes are under consideration. Quite remarkably, the “GigaZ option” in the TESLA Linear Collider project will work at the  $Z$  resonance, reaching a  $Z$  production rate of  $10^9/\text{year}$  [2]. In this way, TESLA could improve by 2 or 3 orders of magnitude the LEP bounds:

$$Br(Z \rightarrow \mu e) < 1.7 \times 10^{-6} \text{ [3]} \quad (1)$$

$$Br(Z \rightarrow \tau \mu) < 1.2 \times 10^{-5} \text{ [3, 4]} \quad (2)$$

$$Br(Z \rightarrow \tau e) < 9.8 \times 10^{-6} \text{ [3, 5]} \quad (3)$$

(with  $\mu e = \mu^+ e^- + \mu^- e^+$ , *etc.*) or perhaps could observe some of these processes.

However, the question arises, whether it is possible to reconcile big lepton-flavor violating  $Z$  decay rates with the stringent experimental bounds on low energy processes, such as:

$$Br(\mu \rightarrow e\gamma) < 1.2 \times 10^{-11} \quad [6] \quad (4)$$

$$Br(\mu \rightarrow 3e) < 1.0 \times 10^{-12} \quad [7] \quad (5)$$

$$R(\mu \rightarrow e \text{ in } {}^{48}_{22}\text{Ti}) < 6.1 \times 10^{-13} \quad [8] \quad (6)$$

We denote with  $R(\mu \rightarrow e \text{ in } N) = \Gamma(N + \mu_s \rightarrow N + e)/\Gamma(\mu_s \text{ capture in } N)$ , where  $N$  is a nuclear species ( $\mu_s$ =stopped muon). In near future, these results will be improved. Indeed, at PSI, a new experiment plans to push  $\mu \rightarrow e\gamma$  down to  $10^{-14}$  [9]; at Brookhaven, the MECO Collaboration aims at a sensitivity better than  $10^{-16}$  [10] for  $R(\mu \rightarrow e \text{ in } \text{Al})$ .

In the present work, we obtain the conditions under which the constraint on  $Z \rightarrow \mu e$  (coming from present  $\mu \rightarrow 3e$  and  $\mu \rightarrow e$  conversion bounds) are weakened. We highlight a special case, that could fall within the TESLA reach. Rather straightforwardly, we first discuss our parametrisation of the  $Z\mu e$  vertex, then describe the generic assumptions we use, and finally study the limits on these form factors (and thence the indirect bounds on  $Z \rightarrow \mu e$ ) coming from the process  $\mu \rightarrow 3e$  and  $\mu \rightarrow e$  conversion in  ${}^{48}_{22}\text{Ti}$ . We finally discuss, in certain specific models, the relation of  $Z \rightarrow \mu e$  with the other lepton-flavor-violating channels  $Z \rightarrow \tau e$  and  $Z \rightarrow \tau \mu$ .

## 2 $Z$ Lepton-Flavor-Violating Form-Factors

The  $Z \rightarrow \mu^+ e^-$  vertex can be parametrized in terms of 6 form factors:

$$V^\alpha = \frac{g}{2c_w} \bar{u}_e(q-p) \left\{ \left( \gamma^\alpha A_1^L + i\sigma^{\alpha\beta} \frac{q_\beta}{M} A_2^L + \frac{q^\alpha}{M} A_3^L \right) P_L + (L \leftrightarrow R) \right\} u_\mu(-p) \quad (7)$$

where  $q$  is the  $Z$  four-momentum,  $g$  is the  $SU(2)_L$  gauge coupling and  $c_w = \cos \theta_w$  ( $\theta_w$ =weak mixing angle). The mass scale  $M$  is introduced to make the form factors  $A_{2,3}^{L,R}$  dimensionless. All form factors, in general, depend on  $q^2$ ; for on-shell  $Z$ , the  $A_3^{L,R}$  form factors do not contribute. Since  $\Gamma_Z^{tot} \simeq 8 \times G_F m_Z^3 / (6\sqrt{2}\pi)$ , one gets for the branching ratio:

$$Br(Z \rightarrow \mu e) \simeq \frac{1}{8} \left( |A_1^L|^2 + \frac{1}{2} \left| \frac{m_Z A_2^L}{M} \right|^2 + (L \leftrightarrow R) \right) \quad (8)$$

To proceed, we have to make certain assumptions. The first one is on the  $q^2$  dependence of the form factors. Indeed, processes like  $\mu \rightarrow 3e$  and  $\mu \rightarrow e$  conversion in  ${}^{48}_{22}\text{Ti}$  probe the form factors at  $q^2 \sim m_\mu^2$  ( $i = 1, 2, 3$ ). So our main assumption is simply that

$$A_i^{L,R}(m_Z^2) \sim A_i^{L,R}(m_\mu^2) \simeq A_i^{L,R} \quad (9)$$

This is expected to happen if the scale of new physics  $M$  responsible for the family lepton numbers violation is bigger than  $m_Z$ . For this reason, we systematically omit the  $q^2$  dependence of the form factors. The second assumption is about cancellations. We will suppose that the experimental constraints are obeyed individually by the  $Z$  exchange contributions, which amounts to assume no major cancellation with other contributions (indeed,  $\mu \rightarrow 3e$  or  $\mu \rightarrow e$  conversion may get contributions, different from  $Z$  bosons exchange).

### 3 Indirect Bounds on $Z \rightarrow \mu e$ from $\mu \rightarrow 3e$

The exchange of a virtual  $Z$  boson leads to  $\mu \rightarrow 3e$  decay. In the limit<sup>1</sup>  $m_e \rightarrow 0$ , and with the definitions (7) we get the rate:

$$\Gamma(\mu \rightarrow 3e) = \frac{G_F^2 m_\mu^5 c_L^2}{24\pi^3} \left\{ \left| A_1^L - \frac{m_\mu A_2^R}{2M} \right|^2 + \frac{1}{2} \left| A_1^R - \frac{m_\mu A_2^L}{2M} \right|^2 + \frac{3}{40} \left| \frac{m_\mu A_2^L}{M} \right|^2 \right\} + (L \leftrightarrow R) \quad (10)$$

where  $c_L = -1/2 + s_w^2$  and  $c_R = s_w^2$  are the usual electron- $Z$  couplings ( $s_w = \sin \theta_w$ ). Using the muon decay rate, and noting that  $s_w^2 \approx 1/4$ , we obtain an useful expression for the  $\mu \rightarrow 3e$  branching ratio, that can be compared with eq. (8):

$$\frac{Br(\mu \rightarrow 3e)}{Br(Z \rightarrow \mu e)} \simeq 6 \times \frac{\left| A_1^L - \frac{m_\mu A_2^R}{2M} \right|^2 + \frac{1}{20} \left| \frac{m_\mu A_2^R}{M} \right|^2 + (L \leftrightarrow R)}{\left| A_1^L \right|^2 + \frac{1}{2} \left| \frac{m_Z A_2^R}{M} \right|^2 + (L \leftrightarrow R)} \quad (11)$$

(note the different masses,  $m_\mu$  and  $m_Z$  in the numerator and in denominator respectively).

Two limiting cases are of particular interests:

1.  $|A_1^L|$  or  $|A_1^R| \gg \frac{m_\mu^2}{M^2} |A_2^{L,R}| \implies Br(\mu \rightarrow 3e)/Br(Z \rightarrow \mu e) \simeq 6$ .

One should emphasize that this case corresponds to most of unified theories where usually the  $Z$  dipole transitions are neglected [11], [12]. The present experimental limit on  $\mu \rightarrow 3e$ , eq. (5), implies that

$$Br(Z \rightarrow \mu e) < 1.7 \times 10^{-13} \quad (12)$$

Of course, if this were the case, there would be no chance to observe the  $Z \rightarrow \mu e$  decay.

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<sup>1</sup>The contributions of  $A_3^{L,R}$  to  $\Gamma(\mu \rightarrow 3e)$  are suppressed by a factor  $m_e$ . They could contribute substantially to this process (but not to  $Z \rightarrow \mu e$ ) if  $A_3/M \sim A_1/m_e$ , namely  $|A_3| > 10^5 \times |A_1|$ , assuming that  $M > m_Z$ . We shall disregard such a possibility.

$$2. \left| A_1^{L,R} \right| \ll \frac{m_\mu^2}{M^2} \left| A_2^{L,R} \right| \implies Br(\mu \rightarrow 3e)/Br(Z \rightarrow \mu e) \simeq \frac{18}{5} \frac{m_\mu^2}{m_Z^2} \simeq 5 \times 10^{-6}.$$

Using again the experimental limit we get<sup>2</sup>

$$Br(Z \rightarrow \mu e) < 2 \times 10^{-7} \quad (13)$$

This indirect bound is considerably weaker than the previous one, but still one order of magnitude better than the direct experimental bound.

This is the case when the bound from  $\mu \rightarrow 3e$  is not incompatible with a large  $Br(Z \rightarrow \mu e)$ , namely within reach for the next generation of colliders like TESLA. For this reason, it will be of particular interest to investigate whether *some* predictive theory or model can fulfil this condition, or if such a fine-tuning for the  $Z$  form factors has other implications.

Similar results have been recently obtained in ref. [13] using considerations based on unitarity.

## 4 Indirect Bounds on $Z \rightarrow \mu e$ from $\mu \rightarrow e$ in Nuclear Field

Let us pass to consider a second interesting process induced by virtual  $Z$  exchange, namely the  $\mu \rightarrow e$  conversion in the nuclear field. In this case, one can use the non-relativistic limit for the nuclear weak current [14], and consider the leading vectorial part. Neglecting the electron mass, one has  $q^0 = p_\mu^0 - p_e^0 \rightarrow 0$ ; once again, the  $A_3^{L,R}$  form factors are expected to give a negligible contribution. If the nucleus is not too heavy ( $A < 100$ ), the leading  $Z$ -contribution to the  $\mu \rightarrow e$  conversion rate is well approximated by [15, 16]:

$$\Gamma(\mu \rightarrow e) = \frac{G_F^2 m_\mu^5}{2\pi^2} \frac{\alpha^3 Z_{eff}^4}{Z} Q_W^2 |F(-m_\mu^2)|^2 \left( \left| A_1^L + \frac{m_\mu A_2^R}{M} \right|^2 + (L \leftrightarrow R) \right) \quad (14)$$

where  $Z$  is its number of protons. The weak charge  $Q_W = Z(1/2 - 2s_w^2) - (A - Z)/2$  in this formula shows that the process is, in first approximation, coherent; the deviations from perfect coherence are quantified by the nuclear form factor  $F(-m_\mu^2)$ , that can be measured by electron scattering [17] ( $|F(-m_\mu^2)| \simeq 0.54$  for  $^{48}_{22}\text{Ti}$  [15]). The parameter  $Z_{eff}$  is the effective atomic charge, obtained by averaging the muon wave function over the nuclear density ( $Z_{eff} \simeq 17.6$  for  $^{48}_{22}\text{Ti}$  [15]). In order to obtain  $R(\mu \rightarrow e \text{ in } ^{48}_{22}\text{Ti})$ , one has to divide (14) by the rate for muon capture. For  $^{48}_{22}\text{Ti}$ ,  $\Gamma(\mu \text{ capture}) = 2.590 \pm 0.012 \cdot 10^6 \text{ s}^{-1}$  [18].

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<sup>2</sup>If  $A_1^{L,R} = m_\mu A_2^{R,L}/2M$ , we can weaken the limit given in eq. (13) by a factor of  $\approx 6$ ; however, in the spirit of our approach (no fine-tuning), this possibility is not stressed.

	$ A_1^L  \text{ or }  A_1^R  \gg \frac{m_\mu^2}{M^2}  A_2^{L,R} $	$ A_1^{L,R}  \ll \frac{m_\mu^2}{M^2}  A_2^{L,R} $
$\mu \rightarrow 3e$	$< 1.7 \times 10^{-13}$	$< 2.0 \times 10^{-7}$
$\mu \rightarrow e \text{ conversion}$	$< 2 \times 10^{-14}$	$< 7 \times 10^{-9}$

Table 1: Indirect bounds on  $Br(Z \rightarrow \mu e)$ , obtained from  $\mu \rightarrow 3e$  and coherent  $\mu \rightarrow e$  conversion. The underlying hypotheses are discussed in the text.

Now, we can compare this partial rate with eq. (8):

$$\frac{R(\mu \rightarrow e \text{ in } {}^{48}_{22}\text{Ti})}{Br(Z \rightarrow \mu e)} \simeq 32 \times \frac{\left|A_1^L + \frac{m_\mu A_2^R}{M}\right|^2 + (L \leftrightarrow R)}{|A_1^L|^2 + \frac{1}{2} \left|\frac{m_Z A_2^R}{M}\right|^2 + (L \leftrightarrow R)} \quad (15)$$

It is particularly interesting to consider two extreme cases:

1.  $|A_1^L| \text{ or } |A_1^R| \gg \frac{m_\mu^2}{M^2} |A_2^{L,R}| \implies R(\mu \rightarrow e \text{ in } {}^{48}_{22}\text{Ti})/Br(Z \rightarrow \mu e) \simeq 32.$

Using the experimental limit given in (6), one gets

$$Br(Z \rightarrow \mu e) < 2 \times 10^{-14} \quad (16)$$

which is one order of magnitude more stringent than eq. (12).

2.  $|A_1^{L,R}| \ll \frac{m_\mu^2}{M^2} |A_2^{L,R}| \implies R(\mu \rightarrow e \text{ in } {}^{48}_{22}\text{Ti})/Br(Z \rightarrow \mu e) \simeq 64 \times \frac{m_\mu^2}{m_Z^2} \simeq 8 \times 10^{-5}.$

Together with the present experimental limit, this implies<sup>3</sup> that

$$Br(Z \rightarrow \mu e) < 7 \times 10^{-9} \quad (17)$$

which should be compared with (1) and with (13). While this limit reduces the hopes to observe the  $Z \rightarrow \mu e$  transition in future colliders, it does not fully exclude an observation at TESLA.

Two remarks are in order: (i) The bounds coming from  $\mu \rightarrow e$  conversion are stronger than the limit coming from  $\mu \rightarrow 3e$ , partially due to the fact that a  $\mu \rightarrow e$  conversion is a coherent effect. (ii) More important, the experimental bound on coherent  $\mu \rightarrow e$  conversion might be soon strengthened by a factor 4 to 7 [19]; indeed, during the last run of the SINDRUM II in 1999, the number of muons stopped was increased by a factor of 4.

For reader convenience, we summarize our results in table 1.

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<sup>3</sup>As noted for  $\mu \rightarrow 3e$ , it is possible to weaken this indirect bound at the price of a fine-tuning: The (leading) coherent contribution can be cancelled if  $A_1^{L,R} = -m_\mu A_2^{R,L}/M$ .

## 5 On the Connection Between $Z \rightarrow \mu e$ , $Z \rightarrow \tau e$ and $Z \rightarrow \tau \mu$

The general approach that we used for  $Br(Z \rightarrow \mu e)$  can be adopted for  $Z \rightarrow \tau e$  and  $Z \rightarrow \tau \mu$ . One has simply to recall that  $\Gamma_\tau \simeq 5 \times (m_\tau/m_\mu)^5 \Gamma_\mu$ . Unfortunately, the experimental bound [20] on processes like  $\tau \rightarrow 3e$  or  $\tau \rightarrow 3\mu$  are much less stringent than on muon decay. In the case of dominance of the vector and axial form factors, one gets

$$\begin{aligned} Br(\tau \rightarrow 3e) &< 2.9 \times 10^{-6} \implies Br(Z \rightarrow \tau e) < 2.5 \times 10^{-6} \\ Br(\tau \rightarrow 3\mu) &< 1.9 \times 10^{-6} \implies Br(Z \rightarrow \tau \mu) < 1.6 \times 10^{-6} \end{aligned}$$

These bounds are slightly better than the direct experimental bounds, eqs. (2) and (3).

However, on theoretical basis, one may expect a stricter relation between the 3 channels of lepton flavor violating  $Z$  decays. Indeed, atmospheric neutrino observations and CHOOZ bounds [21] suggest that the heaviest mass eigenstate has a comparable muon and tau neutrinos component, while electron neutrino is some minor component. Thence, we would expect that the rate of  $Z \rightarrow \mu e$  is comparable to the one of  $Z \rightarrow \tau e$ , while  $Z \rightarrow \tau \mu$  is bigger by some orders of magnitude. Under this view, the results we outlined above would be of more general significance. Let us consider two specific models, in which these considerations can be formalized:

1) Consider the massive neutrino  $\nu_3(x) = \sum_\ell U_{\ell 3} \nu_\ell(x)$  ( $\ell = e, \mu, \tau$ ) that induces atmospheric neutrino oscillations. It has mass  $m_3 \sim 50$  meV, and mixings (=composition in flavor states)  $|U_{\mu 3}| \sim |U_{\tau 3}| \sim 1/\sqrt{2}$  and  $|U_{e 3}| < 0.15$ . Suppose that  $\nu_3$  takes its mass mostly from the coupling with a single right-handed neutrino  $N$  as advocated in [22]:

$$\delta\mathcal{L} = - \left\{ \mu_\ell \times (\overline{N} P_L \nu_\ell) + h.c. \right\} - \frac{M}{2} \overline{N} N;$$

all parameters can be taken real. A modulus-versor decomposition:  $\vec{\mu} = \mu \times (U_{e3}, U_{\mu 3}, U_{\tau 3})$  allows us to relate the parameters of the lagrangian with the properties of the massive neutrino  $\nu_3$ , and to get in particular  $m_3 = \mu^2/M$  (a typical seesaw structure). The crucial point for us is that the mixing between light and heavy neutrino states, namely  $\mu_\ell/M$ , has been related to light neutrino mixings. In this simple model, the form factors are given by  $A_1(Z \rightarrow \mu e) = U_{e3} U_{\mu 3} \times (\mu/M)^2 \times f(M^2/m_Z^2)$  and similar relations,  $f$  being a universal loop function. We get then:

$$\frac{Br(Z \rightarrow \mu e)}{Br(Z \rightarrow \tau e)} = \frac{U_{\mu 3}^2}{U_{\tau 3}^2} \sim 1, \text{ and } \frac{Br(Z \rightarrow \mu e)}{Br(Z \rightarrow \tau \mu)} = \frac{U_{e3}^2}{U_{\tau 3}^2} \sim 2 \times U_{e3}^2 < 0.04 \quad (18)$$

it would seem that the best channel for experimental investigation is  $Z \rightarrow \tau \mu$ , while  $Z \rightarrow \tau e$  should be unobservable. Since the form factors are of the vector or axial type, a 5 orders of

magnitude suppression of  $U_{e3}^2$  would be needed to overcome the limit coming from eq.(16),  $Br(Z \rightarrow \mu e) < 2 \times 10^{-14}$ .

To our knowledge, this is the simplest way to argue for a connection between the rates. However this model fails badly to predict anything measurable, since the *amplitude* is suppressed by  $(\mu/M)^2 = 5 \times 10^{-14} (m_3/50 \text{ meV}) \times (1 \text{ TeV}/M)$ . Note the generality of this (decoupling) feature: The amplitude scales as  $(m_Z/M)^2$  in all models where the lepton-flavor violations are induced by heavy (singlet, right-handed) neutrinos.

2) We then consider a more realistic possibility. This is a supersymmetric  $SU(5) \otimes U(1)_F$  model, where  $U(1)_F$  is the flavor group. The Froggatt-Nielsen mechanism [23] (namely the  $U(1)_F$  selection rules) allows us to explain the mass hierarchies of charged fermions, but also the large  $\nu_\mu - \nu_\tau$  mixing [24]. In a standard scenario, the supersymmetry breaking terms are universal at the grand unification scale  $\Lambda_{GUT}$ ; however, flavour violating effects are induced by the radiative corrections (more details in last paper of [1]). At the electroweak scale, the left-left block of the mass matrix of the scalar leptons gets the contribution:<sup>4</sup>

$$\delta m_{lij}^2 \sim \frac{1}{8\pi^2} (3m_0^2 + A^2) \ln \frac{\Lambda_{GUT}}{M} \epsilon^{2a} \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix}, \quad \text{where } \epsilon = \frac{m_\mu}{m_\tau}$$

$m_0, A$  are the universal supersymmetry breaking mass and trilinear terms (of the order of the electroweak scale) and  $M$  is the average mass of the heavy neutrinos. The Froggatt-Nielsen parameter<sup>5</sup>  $\epsilon$  is:  $\epsilon^2 \simeq 1/300$ ; finally,  $a = 0, 1$  in the cases when the neutrino Yukawa couplings are, respectively, large or small. By mass insertion method, one gets:  $A_1^R(Z \rightarrow \mu e) \simeq \delta m_{l12}^2/m_0^2 \times f(m_0, m_{1/2})$  where  $f$  is a universal loop function, which depends on the sleptons and gaugino masses  $m_0$  and  $m_{1/2}$ . So, one gets:

$$\frac{Br(Z \rightarrow \mu e)}{Br(Z \rightarrow \tau e)} \sim 1, \quad \text{and} \quad \frac{Br(Z \rightarrow \mu e)}{Br(Z \rightarrow \tau \mu)} \sim \epsilon^2 \quad (19)$$

incidentally,  $\epsilon \sim |U_{e3}|$  in these models. Using the indirect bound in (16), we conclude that it is very unlikely to observe  $Z \rightarrow \tau e$  decay at future colliders. But the branching ratio of  $Z \rightarrow \tau \mu$  is just  $\sim \epsilon^{-2}$  larger; thus, using the value of  $\epsilon$  quoted above, we get  $Br(Z \rightarrow \tau \mu) \lesssim 6 \times 10^{-12}$ . Even with a generous allowance of coefficients order unity, this limit remains stringent; thus, this model suggests that lepton flavor violating  $Z$  decays are not within reach.

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<sup>4</sup>To correctly interpret these equations, one should recall the presence of not-spelled “coefficients order unity”, that however are not expected to change the order-of-magnitude estimations. Similar results can be obtained in other unified models, as  $SU(3)_c \otimes SU(3)_L \otimes SU(3)_R \otimes U(1)_F$  [25]—see eq. (38) therein.

<sup>5</sup>Other values of  $\epsilon$  are motivated and discussed in [26].

## 6 Summary and Discussion

1. We obtained the connection between  $Z \rightarrow \mu e$  and  $\mu \rightarrow 3e$ , namely eq. (11). We have shown that if the axial and vector  $Z$  form factors are dominant,  $Br(Z \rightarrow \mu e)$  is far away from the sensitivity of future colliders. Conversely, if the  $Z$  dipole transitions are the dominant ones, the constraint from  $\mu \rightarrow 3e$  is much weaker, and  $Br(Z \rightarrow \mu e)$  can be as large as  $10^{-7}$ .
2. The indirect bound on  $Br(Z \rightarrow \mu e)$  from coherent  $\mu \rightarrow e$  conversion (from eq. (15)) is however stronger by more than 1 order of magnitude (see again table 1). In the most optimistic assumption (=dominance of dipole form factors), the experimental bound on coherent  $\mu \rightarrow e$  conversion yields the stringent indirect bound  $Br(Z \rightarrow \mu e) < 7 \times 10^{-9}$ , which may be improved soon by a factor of 4 – 7.
3. These indirect bounds are valid *modulo* very specific cancellations between various contributions (or perhaps involving photon-exchange, box diagrams, or exotics)—see eq. (9) and discussion therein, and footnotes 2 and 3.
4. Our model-independent analysis suggests a theoretical challenge, namely the construction of a model (theory) where the dipole form factors are large. Such a hypothetical model would be rather interesting in connection with lepton flavor violating processes, and in particular with  $Z \rightarrow \mu e$ .
5. We have discussed the relation among the decay channels  $Z \rightarrow \mu e$ ,  $Z \rightarrow \tau e$  and  $Z \rightarrow \tau \mu$  in certain specific models. Phenomenological and theoretical arguments (eqs. (18,19)) point to a simple scaling law:

$$Br(Z \rightarrow \mu e) : Br(Z \rightarrow \tau e) : Br(Z \rightarrow \tau \mu) \sim 1 : 1 : U_{e3}^{-2}$$

We expect that such a relation holds for models where the sources of lepton flavor violations are tightly connected with neutrino masses.

6. We conclude by remarking on the implications of previous relation, conjecturing its validity. Let us assume that vector and axial  $Z$  form factors dominate. Due the indirect bounds on  $Z \rightarrow \mu e$  discussed in the present work, a positive signal  $Z \rightarrow \tau \mu$  would be related to very small values of  $U_{e3}^2 < 10^{-5}$ . The decay  $Z \rightarrow \tau e$ , instead, would be certainly too small to be observable.



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